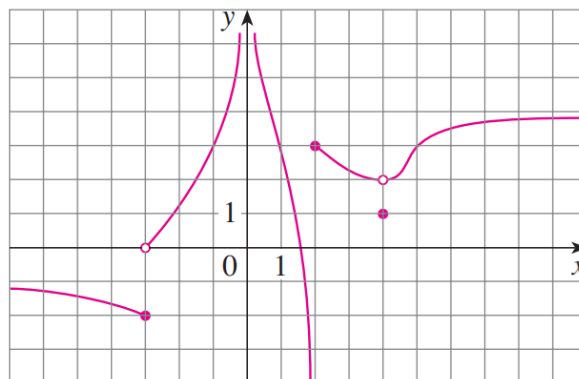


Exercise 1

The graph of f is given.



(a) Find each limit, or explain why it does not exist.

$$(i) \quad \lim_{x \rightarrow 2^+} f(x) \quad (ii) \quad \lim_{x \rightarrow -3^+} f(x) \quad (iii) \quad \lim_{x \rightarrow -3} f(x)$$

$$(iv) \quad \lim_{x \rightarrow 4} f(x) \quad (v) \quad \lim_{x \rightarrow 0} f(x) \quad (vi) \quad \lim_{x \rightarrow 2^-} f(x)$$

$$(vii) \quad \lim_{x \rightarrow \infty} f(x) \quad (viii) \quad \lim_{x \rightarrow -\infty} f(x)$$

(b) State the equations of the horizontal asymptotes.

(c) State the equations of the vertical asymptotes.

(d) At what numbers is f discontinuous? Explain.

Solution

Calculate the limits using the given graph.

$$(i) \quad \lim_{x \rightarrow 2^+} f(x) = 3$$

$$(ii) \quad \lim_{x \rightarrow -3^+} f(x) = 0$$

$$(iii) \quad \lim_{x \rightarrow -3} f(x) = \text{does not exist because left- and right-hand limits are unequal}$$

$$(iv) \quad \lim_{x \rightarrow 4} f(x) = 2$$

$$(v) \quad \lim_{x \rightarrow 0} f(x) = \infty$$

$$(vi) \quad \lim_{x \rightarrow 2^-} f(x) = -\infty$$

$$(vii) \quad \lim_{x \rightarrow \infty} f(x) = 4$$

$$(viii) \quad \lim_{x \rightarrow -\infty} f(x) = -1$$

The horizontal asymptotes are

$$y = 4 \quad \text{and} \quad y = -1,$$

the vertical asymptotes are

$$x = 0 \quad \text{and} \quad x = 2,$$

and f is discontinuous at

$$x = -3 \quad \text{and} \quad x = 0 \quad \text{and} \quad x = 2 \quad \text{and} \quad x = 4.$$